Problem on ISA

(1) Calculate the std atm value of T, P, ρ at a geo-potn alt of 14 Km.

$h = 14\ Km$

From fig $h = 14\ Km$ falls in Isothermal region

so $T = 216.66\ K$

Find P & ρ

To find P & ρ along the isothermal rgn, we need to have the value of P & ρ along the gradient layer. This is bcoz, we need to know the base value to cal. & grad rgn forms the base layer for the Iso therm rgn.

P in grad rgn

First find at $h = 11\ Km$

$$\frac{P}{P_1} = \left(\frac{T}{T_1}\right)^{\frac{-g_0}{aR}}$$

$$P = P_1 \left(\frac{T}{T_1}\right)^{\frac{-g_0}{aR}}$$

$$P = 1.01 \times 10^5 \left(\frac{216.66}{288.16}\right)^{\frac{-9.8}{0.00625 \times 287}}$$

$$\boxed{P = 2.26 \times 10^4\ N/m^2} \rightarrow \text{At } h = 11\ Km$$

$$\rho = \rho_1 \left(\frac{T}{T_1}\right)^{-\left(\frac{g_0}{aR}+1\right)}$$

$$\rho = 1.23 \left(\frac{216.66}{288.16}\right)^{-\left(\frac{-9.81}{-0.00645 \times 287}+1\right)}$$

$$\boxed{\rho = 0.367\ kg/m^3} \rightarrow \text{At } h = 11\ Km$$

$R = 8.314\ J/mol\ K$

for air, One mole $= 28.97g$ or $0.02897\ kg$.

$$R = \frac{8.314}{0.02897} = 287\ J/kg\ K$$

Now P & ρ are the base value for the isothermal region.

$P_1 = 2.26 \times 10^4 \ N/m^2$

$\rho_1 = 0.367 \ kg/m^3$

for h = 14 Km

$P = P_1 e^{-(g_0/RT)(h-h_1)}$

$P = 2.26\times10^4 \times e^{\left(\frac{-9.81}{287(216.66)}\right)(14000-11000)}$

$P = 1.41\times10^4 \ N/m^2$

||ly

$\frac{P}{P_1} = \frac{\rho}{\rho_1}$

$\rho = \rho_1\left(\frac{P}{P_1}\right) = 0.367 \times \left(\frac{1.41\times10^4}{2.26\times10^4}\right)$

$\rho = 0.23 \ kg/m^3$

Addn problem

A Supersonic fight a/c is cruising at an altitude of 28Km from the sea level. Det the atm press sensed by the a/c at that geo potn alt of 28Km.

20) At 12 Km in the std atm, the pres, ρ & Temp are $1.9399\times10^4 \ N/m^2$, $3.1194\times10^{-1} \ kg/m^3$ & 216.66K resp. Using these values, cal the std atm values of P, ρ & T at an alt of 18Km & check with the std altitude table.

Consider 12 Km as the base & find at 18Km in Iso thermal layer.

$\frac{P_2}{P_1} = e^{-(g_0/RT)(h_2-h_1)}$

$P_2 = 1.9399\times10^4 \times e^{-\frac{9.81}{287\times216.66}\times6000}$

$P_2 = 7.53\times10^3 \ N/m^2$

P_{18Km} –

ρ_{18Km} –

||ly $\rho_2 = 0.121 \ kg/m^3$

(3) At what value of the geometric altitude is the diff $h - h_g$ equal to 2% of h?

A) $|h - h_g| = 2\% \, h$

$\frac{|h - h_g|}{h} = 0.02$

$\left|1 - \frac{h_g}{h}\right| = 0.02.$

Initially the diff b/w h & hg is less than 1%, especially upto 65 Km. After 65Km, the diff b/w h & hg becomes slightly greater than 1. As the alt ↑ the diff ↑ slightly.

$\frac{h_g}{h} = \frac{r + h_g}{r}$

$\left|1 - \left[\frac{r + h_g}{r}\right]\right| = 0.02.$

$\left|\frac{r - r \mp h_g}{r}\right| = 0.02$

$h_g = 0.02 r$

$h_g = 0.02 (6.36 \times 10^6)$

$h_g = 127 \text{ Km}$

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(4) An F-16 hypersonic fighter a/c is in a rapid climb. At the instant, it passes thro a std alt of 8000m, it time rate of change of altitude is 150 m/s (rate of climb). Corres to this rate of climb at 8000m is a time rate of change of ambient psr. Cal this rate of chay of psr.

→ Since 8000m falls within the first gradient rgn in the std atm. Here the varn of psr & temp are given by,

$$\frac{P}{P_1} = \left(\frac{T}{T_1}\right)^{\frac{-g}{aR}} \quad (1)$$

$$T = T_1 + a(h - h_1) \quad (2)$$

Diff (1) w.r.t. time

$$\frac{P}{P_1} = \left(\frac{T}{T_1}\right)^{-\frac{g}{aR}}$$

$$\frac{1}{P_1}\frac{dP}{dt} = \left(\frac{1}{T_1}\right)^{-\frac{g}{aR}} \frac{d}{dt}\left[T\right]^{-\frac{g}{aR}}$$

$$\frac{1}{P_1}\frac{dP}{dt} = \left(\frac{1}{T_1}\right)^{-\frac{g}{aR}} \frac{-g}{aR}\left[T\right]^{\frac{-g}{aR}-1} \frac{dT}{dt} \quad -(3)$$

$$T = T_1 + a(h - h_1)$$

diff w.r.t. time

$$\frac{dT}{dt} = \cancel{T_1}^{\to 0} + ah - \cancel{ah_1}^{\to 0}$$

$$\frac{dT}{dt} = a\frac{dh}{dt} \quad -(4)$$

Sub (4) in (3)

$$\frac{dP}{dt} = P_1 \left(T_1\right)^{\frac{g}{aR}} \left(\frac{g}{\cancel{a}R}\right) T^{-\left(\frac{g}{aR}+1\right)} \cancel{a}\frac{dh}{dt}$$

$$\boxed{\frac{dP}{dt} = -P_1 \left(T_1\right)^{\frac{g}{aR}} \left[\frac{g}{R}\right] T^{-\left[\frac{g}{aR}+1\right]} \frac{dh}{dt}}$$

$\frac{dh}{dt} \to$ rate of climb = ~~[illegible]~~ 150 m/s

$a = -6.5 \times 10^{-3}$ °K/km $\to$ first gradient region

At sea level

$P_1 = 1.01325 \times 10^5$ N/m², $T_1 = 288.18$ K, $T_{8000m} = 236.23$ K from table

$R = 287$ J/kgK. $g = 9.81$ m/s²

(7)

$$\frac{dp}{dt} = -1.01325\times10^5 \times (288.18)^{\frac{9.81}{-0.00625\times287}} \times \left[\frac{9.81}{287}\right] (236.23)^{-\left[\frac{9.81}{-aR}+1\right]} \times 150$$

$$= -1.01325\times10^5 \times 288.18^{-5.46} \times 0.03418 \times (236.23)^{+6.468} \times 150$$

(3.83×10^{-14}) $\quad$ $4.45\times10^{+15}$

$$\boxed{\frac{dp}{dt} = \_\_\_\_\_\_ \frac{N}{m^2\,sec}}$$